On the Behavior of Friction in Lubricated Point Contact With Provision for Surface Roughness

This paper presents a simple approach to predict the behavior of friction coefficient in the sliding lubricated point contact. Based on the load-sharing concept, the total applied load is supported by the combination of hydrodynamic film and asperity contact. The asperity contact load is determined in terms of maximum Hertzian pressure in the point contact while the fluid hydrodynamic pressure is calculated through adapting the available numerical solutions of elastohydrodynamic lubrication (EHL) film thickness formula for smooth surfaces. The simulations presented cover the entire lubrication regime including full-film EHL, mixed-lubrication, and boundary-lubrication. The results of friction, when plotted as a function of the sum velocity, result in the familiar Stribeck-type curve. The simulations are verified by comparing the results with published experimental data. A parametric study is conducted to investigate the influence of operating condition on the behavior of friction coefficient. A series of simulations is performed under various operating conditions to explore the behavior of lift-off speed. An equation is proposed to predict the lift-off speed in sliding lubricated point contact, which takes into account the surface roughness. [DOI: 10.1115/1.4000306]

1 Introduction

Research in elastohydrodynamic lubrication (EHL) study added a new dimension when researchers turned their attention from rectangular to elliptical conjunctions in the early 1970s. This began in 1970 when Cheng [1] developed a Grubin type inlet analysis applicable to elliptical Hertzian contact. Later, Hamrock and Dowson [2] conducted a comprehensive study on EHL analysis of ball bearings where they showed that the theoretical features are in overall agreement with experiments. More recently, computational schemes that utilize multilevel solvers are extended for prediction of film thickness in lubricated elliptic contacts. The pressure and film thickness on the center-line of the contact are predicted accurately from an equivalent point contact analysis [3].

EHL research has progressively continued. Yet most of the published works are limited to “smooth” interacting surfaces where roughness and surface features are absent in the formulation. In fact, the classical analysis of elastic contact between two spheres as formulated and solved by Hertz pertains to smooth surfaces while engineering surfaces are rough. Therefore, when two mating surfaces are pressed together, most of the real contact occurs at the tips of the surface asperities. The asperities of mating surfaces behave like a spring as they interact under an applied load, so that contact is extended over a larger area than the contact between two smooth surfaces. Under a high applied load, the contact region in the central area is surrounded by a fringe in which only higher asperities are involved in contact. On the other hand, a reduction in the real contact area may occur due to light applied load where the asperities are not flattened; this may result in an increase in contact pressure due to asperity peak interaction of a rough surface. It is clear that the classical Hertzian analysis cannot hold for describing the contact of rough surfaces.

The elastic contact of rough spheres has been studied by several investigators. Greenwood and Tripp [4] extended the Hertzian theory of elastic contact to a number of discrete microcontacts in an attempt to find the real pressure distribution over the contact area. Yip and Venart [5] formulated the contact between curved rough bodies based on the assumptions that the roughness height distribution is Gaussian and that the deformation of the surface roughness is elastic while that due to waviness is fully plastic.

Zhu and Ai [6] presented a numerical solution for the EHL analysis of point contacts where they investigated the surface roughness effects on the average film thickness and the pressure peak. Hu and Zhu [7] proposed a new, simple, and robust numerical approach for the mixed EHL analysis of point contact. The system of equations and the numerical procedure are unified for a full coverage of all the lubrication regions including the full-film-, mixed-, and boundary-lubrications. A comprehensive study considering thermal effects is presented by Wang et al. [8]. This model deterministically calculates pressure and surface temperature by simultaneously solving a system of equations that govern the lubrication, elastic deformation, and thermal behaviors of a point contact. Simulations and measurements of sliding friction coefficient between rough surfaces in point contacts are performed by Wang et al. [9]. They used a full-scale deterministic solution of mixed-lubrication for the prediction of film thickness, pressure, and the area of real contact. The friction from asperity contact was determined in terms of the limiting shear stress while the fluid shear stress in the lubrication areas was calculated using different rheological models. Their simulations covered the entire lubrication regime including full-film EHL, mixed-lubrication, and boundary-lubrication. The results for friction plotted as a function of sliding velocity revealed a Stribeck-type behavior.

Most of the recent available efforts for predicting friction behavior in lubricated point contact require solving the Reynolds, deformation, load balance, and appropriate rheology equations simultaneously using an iterative procedure that is generally time consuming and often experiences computational sensitivity. The need for a simple and robust model capable of rapidly predicting the behavior of rough point contact still remains. To fulfill this need, a deterministic model for treating point contact problems is presented that implements the load-sharing concept pioneered by Johnson et al. [10]. The idea was first proposed by Gelincik and Schipper [11] in sliding lubricated line contact and then continued by Liu et al. [12] who investigated the effects of rheological behavior of the lubricant on the friction coefficient. Lu et al. [13] extended the idea further with experimental verification.
and Schipper [14] applied this method to consider the starvation effect in lubricated line contact. A follow up study is performed by Akbarzadeh and Khonsari [15] who applied Johnson’s load-sharing concept to lubrication analysis of gears. All of these publications are limited to the line contact.

Liu et al. [16] predicted the friction in point contact conjunctions based on load-sharing concept; however, their study is limited to heavy load operating conditions. They assumed that the asperity contact pressure is equal to Hertzian point contact pressure, which could be a rough estimation in highly loaded conditions. The objective of this study is to extend the EHL line contact method developed in Refs. [11–14] to sliding lubricated point contact and to provide a simple model, which is useful for control applications where rapid prediction and friction compensation are essential for instruments supported on ball bearings. For this purpose, a series of curve fits is developed that can be used for approximating pressure and real area of contact; the essential point is that the behavior of rough surfaces is determined assuming Gaussian statistical distribution of asperity heights subject to elastic deformation. Bair–Winer [17] model is adopted to describe the shear stress of the lubricant. The model and the solution algorithm are capable of predicting the friction behavior of lubricated rough point contact surfaces. The model is simple, but realistic and easily converges to the desired results. Rapid convergence and computational time for the prediction of friction coefficient in the entire range of operation are the beauty of the proposed model, making it useful in real-time applications.

2 Theoretical Development

2.1 Friction Expression. Based on Johnson’s load-sharing concept, the total applied load 

\[ F_T = F_H + F_C \] (1)

Applying the load-sharing concept, the problem of solving the Reynolds, deformation, load balance, and rheological equations simultaneously can be easily replaced with the problem of the dry point contact and the problem of lubricated point contact. Using the relevant scaling factor for the hydrodynamic part \( \gamma_1 \) and asperity contact \( \gamma_2 \), Eq. (1) can be rewritten as

\[ F_T = \frac{F_T}{\gamma_1 \gamma_2} \] (2)

Similarly, the total friction force is the sum of two components

\[ F_f = F_{f,H} + F_{f,C} \] (3)

where \( F_{f,H} \) is the hydrodynamic friction force given by the following expression:

\[ F_{f,H} = \int \tau_{dl} dA_H \] (4)

where \( \tau_{dl} \) represents the fluid shear stress and \( A_H \) is the contact area of the fluid.

The asperity contact friction is given by

\[ F_{f,C} = \sum_{i=1}^{N} \int \tau_{Ci} dA_{Ci} \] (5)

where \( \tau_{C} \) represents the contact shear stress of an individual asperity, \( A_{Ci} \) is the area of an individual asperity in contact, and \( N \) is the number of the asperities in contact.

The asperity friction may be written as the product of an average asperity friction coefficient \( f_c \) and the load carried by the asperities \( F_{C} \). Therefore, we arrive at the following relation for the friction force:

\[ F_{f,C} = f_c F_C \] (6)

To determine the shear stress of hydrodynamic films, the traction model developed by Bair and Winer [17] is adopted. It is defined as

\[ \eta' = \frac{\eta}{\lambda} \] (7)

where

\[ \lambda = \frac{\eta' \sqrt{\gamma}}{\tau_e} \] (8)

where \( \gamma \) is the shear strain rate and \( \tau_e \) is the limiting shear stress, which is considered as a function of pressure described by

\[ \tau_e = \tau_{el} + \beta_p p_a \] (9)

where \( \tau_{el} \) is the limiting shear stress at the ambient pressure, \( \beta_p \) is the slope of the limiting shear stress-pressure relation, and \( p_a \) is the mean pressure of Hertzian point contact.

The fluid traction force per unit length \( F_f \) is determined by integrating the shear stress

\[ F_f = \int \tau' dx = \int \eta' dy dx \] (10)

Assuming the separation of two rough surfaces is constant and equal to central film thickness \( h_{c} \), the fluid traction force then can be rewritten as

\[ F_{f,H} = \tau_{f_1}(1 - e^{-\eta' h_{c}/\tau_e}) \cdot \pi a^2 \] (11)

where \( a \) is the effective velocity and \( a \) is the radius of point contact. According to Roelands’s formula [18], the lubricated viscosity up to a certain elevated pressure is given by

\[ \eta = \eta_b \left( 1 + \frac{p_v}{\eta_b} a^2 \right) \] (12)

where \( \eta_b \) is the oil viscosity at inlet temperature and \( \eta_b \) is 6.35 \times 10^{-5} \text{ Pa s}. The parameter \( Z \) represents the pressure-viscosity index as [19]

\[ Z = \frac{\alpha}{[5.1 \times 10^7 (\ln \eta_b + 9.67)]} \] (13)

where \( \alpha \) is the pressure-viscosity coefficient.

Another expression for the change in viscosity with pressure, often attributed to Barus [20], is of the following form:

\[ \eta = \eta_b e^{\nu p_a} \] (14)

Hence, the total friction coefficient can be obtained from

\[ f = \frac{F_f}{F_T} = \frac{F_{f,H} + F_{f,C}}{F_T} \] (15)

2.2 Governing EHL Equations. There are three basic equations for the EHL problem: Reynolds equation, the deformation equation, and the load balance. Nijenhout et al. [3] developed an accurate relationship to predict the central film thickness based on numerical solution of the following equations:

\[ \bar{h}_{c} = \left( \bar{h}_{R} + \bar{h}_{E}^{-4} + \bar{h}_{F}^{-4} \right)^{-3/8} \bar{h}^{3/2} + \bar{h}_{R}^{-8} + \bar{h}_{E}^{-8} \bar{h}^{3/2} \] (16)

where

\[ s = 1.5(1 + e^{-1.25 \bar{h}_{RI}^{1/3}}) \] (17)

with the following group of dimensionless parameters:

\[ \bar{h}_{RI} = 1.8D^{1/2}, \quad \bar{h}_{EI} = C_{RI} \bar{M}^{2}, \quad C_{RI} = 145(1 + 0.796D^{1/4} \bar{M}^{-1})^{-15/3} D^{-1} \] (18)
The surface. The bulk compression is elastic-piezoviscous. Greenwood and Tripp [21] defined as follows:

\[ \tilde{h}_{EP} = \frac{C_{EP} M^{-2/3}}{C_{EI}} = 3.18(1 + 0.006 \ln D + 0.63 D^{2/5})^{-1/2} D^{-1/2} \]

The additional dimensionless parameters will be defined as follows:

\[ M = WU_x^{-3/4}, \quad L = GU_x^{-1/4}, \quad W = \frac{F}{E' R_x}, \quad G = aE', \]

\[ U_x = \frac{\eta \mu}{E' R_x}, \quad \tilde{h}_c = \tilde{h}_{cen} U_x^{-1/2}, \quad \tilde{h}_{cen} = h/R_x \]

(19)

The subscripts denote the following: RI is rigid-isoviscous, RP is rigid-piezoviscous, EI is elastic-isoviscous, and EP is elastic-piezoviscous.

In the mixed-lubrication regime the total pressure \( p_T \) is split into hydrodynamic pressure \( p_H \) and asperity contact pressure \( p_C \). Therefore, the film thickness should be appropriately implemented according to Eqs. (2) and (3). This is done by writing \( p_T = \gamma_H p_H + \gamma_C p_C \). In Reynolds equation the pressure \( P \) is replaced with \( p_H \) whereas in deformation and load balance equations the total pressure is used. Replacing \( E' \) with \( E'/\gamma_1 \) and \( F_T \) with \( F_T/\gamma_1 \), the central film thickness for the mixed-lubrication in a point contact is expressed as

\[ \tilde{h}_c = [(\tilde{h}_{EI})^{3/2} \gamma_1 + (\tilde{h}_{EP})^{3/2} \gamma_1^{-2/5} + (\tilde{h}_{RP})^{-4/5}]^{2/3} + [(\tilde{h}_{EP})^{-4/5} \gamma_1 + (\tilde{h}_{RP})^{-5/4}]^{1/3} \times \gamma_1^{1/2} \]

(20)

where

\[ s = 1.5(1 + \epsilon^{-1} 2 \gamma_1^{-7} \tilde{h}_{EP} \gamma_1) \]

(21)

2.3 Asperity Contact Term. Quantitative analysis of dry point contact using the Greenwood and Williamson [21] model of a rough surface is applied to the point contact of spheres by Greenwood and Tripp [4]. The axisymmetric case can be simplified to the contact of smooth sphere of radius \( R \) with a nominally flat rough surface having a standard distribution of summit heights \( \sigma_e \). The separation \( d \) between two rough, deformed surfaces is given by

\[ d(r) = w_b(r) - y(r) = -y_0 + (r^2/2R) + w_b(r) \]

(22)

where \( w_b \) is the bulk displacement, \( y(r) \) is the profile of the undeformed sphere relative to the data, and \( y_0 \) is its value at the center of the contact. Datum is taken at the mean level of the rough surface. The bulk compression \( w_b \) can be written as [4]

\[ w_b(r) = \frac{4}{\pi E'} \int_0^a \frac{s}{s + r} p(s) K(k) ds \]

(23)

where \( K(k) \) is the complete elliptic integral of the first kind with argument \( k = 2(2s)^{1/2}(r+s) \). Assuming that the asperities deform elastically, the effective pressure at radius \( r \) is found to be

\[ p(r) = \frac{4}{\sqrt{\pi} E'} \int_0^\infty (z_s - d(r))^{3/2} e^{-\epsilon z_s^2} ds \]

(24)

where \( z_s \) is the height of the asperity summit above the data. Equations (22)–(24) are solved by Greenwood and Tripp [4] for the case of asperity heights with Gaussian distribution. Central film thickness is used to obtain the pressure in the surface asperities. In the present study, a series of curve fits is developed to relate the maximum asperity contact pressure, which occurs at the center of the point contact to the maximum Hertzian point contact. The result is given as follows:

\[ \frac{p_c}{p_0} = a + be^{-\epsilon} + \frac{c \ln(\mu)}{\mu} \]

(25)

The dimensionless parameters used in Eq. (25) are defined as

\[ \xi = \sigma_e \left( \frac{16 E' h_{EI}^2}{9 F_T} \right)^{1/3} \]

(26)

\[ \mu = \frac{8 \pi r_0 \sigma_e (2R \beta)^{1/2}}{7} \]

(27)

where \( a = 0.2342, b = 0.8189 \), and \( c = -0.2720 \), and \( p_0 \) represents the maximum Hertzian pressure for the point contact

\[ p_0 = \frac{3 F_T h_{EI}^2}{2 \pi R^2} \]

(28)

It is important to note that the sliding velocity affects the influence of deformed asperities in EHL contact; this effect is ignored in the present study. The results of the calculation for the asperity contact are adopted for the mixed-lubrication analogous to the hydrodynamic term. Thus, \( p_T = \gamma_H p_H + \gamma_C p_C \). The results for the mixed-lubrication then can be obtained by replacing \( E' \) with \( E'/\gamma_1 \), \( F_T \) with \( F_T/\gamma_1 \), and \( n \) with \( n \gamma_0 \). For the central pressure in a rough point contact these substitutions in Eqs. (25), (28), and (29) yield the following result:

\[ F_{3/2} \left( \frac{h(s)}{\sigma_e} \right) = \frac{1}{\sqrt{\pi} \int_{h_0}^{h_0 + y} \left( h(s) - \sigma_e \right)^{3/2} e^{-(s/\sigma_e)^2} ds \}

(30)

with the following group of dimensionless parameters:

\[ \bar{F}_T = \frac{\pi R^2 h_{EI}^{1/3}}{6 F_T} \]

\[ \bar{n} = n R \sqrt{\beta}, \quad \bar{\sigma}_e = \frac{\sigma_e}{R} \]

\[ \bar{h}_c = \frac{h_{cen}}{R}, \quad \bar{d}_c = \frac{d}{R} \]

(32)

3 Solution Procedure

There are three unknowns \((\gamma_1, \gamma_2, \text{ and } h_c)\) associated in Eqs. (2), (20), and (31). An iteration method is utilized to solve this set of nonlinear equations. For a given steady velocity and load, an initial value for \( \gamma_2 \geq 1 \) is assumed and \( \gamma_1 \) is obtained using Eq. (2); then Eq. (20) is solved for the central film thickness \( h_c \). Having the value for the central film thickness, the validity of Eq. (31) is checked. The iterations are continued until the error between two successive iterations falls below a specified tolerance value (\( \epsilon_T = 0.001 \)). This procedure is performed for the next sliding velocity. In the following simulations, general relations of film thickness for the contact of elliptical surfaces are reduced to the contact of spheres assuming \( D = 1 \) and \( R = R_e = R \) results in \( k = 1 \). The computations are typically converged within 10 s predicting the Strubeck curve under given operating conditions.
4 Results and Discussion

4.1 Verification. In order to verify the prediction of the model for the friction coefficient, we use the published experimental results by Wang et al. [9]. Their setup consists of a rotary flat sample in contact with a steel ball, which has a diameter of 25.4 mm. The contact zone is immersed in a lubricant reservoir. Two different sets of experiments were conducted: reciprocal mode and rotary mode. In this study, only the second category of the results is used for validation purposes. The tests were started from a low speed and then increased in a stepwise manner until the maximum speed was reached. At each speed the test lasted for 2 min to ensure that the steady-state condition is reached. The simulation is performed within the same speed range of the experiments reported in Table 3 of Ref. [9]. The input parameters for the simulations are reported in Table 1; these correspond to experimental results reported by Wang et al. According to Ref. [9], the experiments were repeated to investigate the reproducibility of the results. Four friction curves are measured on four samples whose \( \sigma_r \) have the same value of 1.6 \( \mu m \). The density of asperities \( n \) and the average radius for the asperities \( \beta \) for the simulation are chosen so that \( \Phi = \sigma_r \rho \beta \) is 0.032 and the best agreement with the experiments is reached. According to Greenwood and Williamson [21], for engineering surface, \( \Phi \) typically varies between 0.03 and 0.05. It is also assumed that the Coulomb friction coefficient \( f_c \) for the asperity contact is 0.16.

Figure 1 shows the simulation result and the comparison with experiments reported in Ref. [9]. The simulation results are in good agreement with the experimental results of Wang et al. (Fig. 7 of Ref. [9]). The model predicts that the friction coefficient drops in the mixed-lubrication regime and increases slightly after the transition from mixed- to full-film-lubrication regime. The discrepancies between the simulations and experimental results can be, in part, attributed to simplifications in the model and the uncertainty in some of the input variables used in the simulations.

According to Ref. [9], while the overall trend remains the same, the inconsistency in samples each made individually in the machine shop results in different magnitudes for friction coefficient. Two different values are used in simulations; comparison of the results reveals that a change in the asperity friction coefficient results in a change in the predicted friction coefficient in the boundary- and mixed-lubrication regimes. Deviation of the friction coefficient becomes significant while approaching the boundary regime from the mixed-lubrication regime. In the full-film lubrication regime, the contribution of asperities is nil and the two curves coincide.

4.2 Film Thickness Parameter. Figure 2 examines the variation of the film thickness parameter \( \Delta (A = h_c / \sigma_r) \) with velocity. The input parameters of the simulation are tabulated in Table 2. The results reveal that an increase in the film thickness parameter in the mixed-lubrication regime results in a decrease in the friction coefficient while there is a reverse influence in the full-film lubrication regime. The film thickness parameter increases exponentially while the transition from the mixed- to the full-film lubrication occurs.

The effect of pressure-viscosity relations on the total friction coefficient is also investigated. Roelands [18] and Barus’s [20] pressure-viscosity relation is represented by Eqs. (12) and (14), respectively. As a result, the calculated friction coefficient in the boundary- and mixed-lubrication regimes is nearly identical using each of these rheological models. Near the transition from mixed to the hydrodynamic regime, Barus’s model predicts a higher friction coefficient and that this trend continues in the hydrodynamic regime. Using these pressure-viscosity relations, the same trend is reported by Wang et al. [9]. It should be noted, however, that these

---

Table 1 Input parameters for steady simulations and experiments [9]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n )</td>
<td>( 1 \times 10^{10} ) m(^{-2} )</td>
</tr>
<tr>
<td>( \beta )</td>
<td>( 2 \times 10^{-6} ) m</td>
</tr>
<tr>
<td>( \sigma_r )</td>
<td>( 1.6 \times 10^{-6} ) m</td>
</tr>
<tr>
<td>( E' )</td>
<td>( 2.19 \times 10^{11} ) Pa</td>
</tr>
<tr>
<td>( \eta_0 )</td>
<td>0.1756 Pa s</td>
</tr>
<tr>
<td>( R )</td>
<td>0.0127 m</td>
</tr>
<tr>
<td>( f_c )</td>
<td>0.16</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>( 18.8 \times 10^{-9} ) Pa(^{-1} )</td>
</tr>
<tr>
<td>( \beta_0 )</td>
<td>0.05</td>
</tr>
<tr>
<td>( \tau_{LO} )</td>
<td>( 2 \times 10^{6} ) Pa</td>
</tr>
</tbody>
</table>

Table 2 Input parameters for parametric study simulations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n )</td>
<td>( 5 \times 10^{10} ) m(^{-2} )</td>
</tr>
<tr>
<td>( \beta )</td>
<td>( 10 \times 10^{-6} ) m</td>
</tr>
<tr>
<td>( \sigma_r )</td>
<td>( 0.05 \times 10^{-6} ) m</td>
</tr>
<tr>
<td>( E' )</td>
<td>( 2.19 \times 10^{11} ) Pa</td>
</tr>
<tr>
<td>( \eta_0 )</td>
<td>( 8 \times 10^{-3} ) Pa s</td>
</tr>
<tr>
<td>( R )</td>
<td>0.01 m</td>
</tr>
<tr>
<td>( f_c )</td>
<td>0.15</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>( 9.4 \times 10^{-9} ) Pa(^{-1} )</td>
</tr>
<tr>
<td>( \beta_0 )</td>
<td>0.047</td>
</tr>
<tr>
<td>( \tau_{LO} )</td>
<td>( 2.5 \times 10^{6} ) Pa</td>
</tr>
</tbody>
</table>

---

Fig. 1 Variation of the total friction coefficient (present simulations and experimental results [9])

Fig. 2 The film thickness parameter and effect of pressure-viscosity relations on the total friction coefficient, \( F_c = 40 \) N and \( \eta_0 = 0.01 \) Pa s
relationships best describe the viscosity at moderate values of pressures.

The results of a series of simulations are presented in Secs. 4.3–4.5 to investigate the influence of key operational parameters in a point contact configuration.

4.3 Effect of Load. Figure 3 shows the simulation results for the sliding lubrication of a smooth ball in contact with a rough flat surface. The curves for different applied loads show the deviation in the friction coefficient in three different regimes: boundary, mixed, and full. The physical explanation of the problem is interesting. In the boundary regime, increasing the applied load forces more asperities into contact; this results in higher coefficient of friction. The film thickness increases due to an increase in the sliding velocity when the operating regime initiates from the boundary-lubrication, passing through the mixed-lubrication, and ending in the full-lubrication.

The total friction coefficient is the combination of asperity contact friction and hydrodynamic friction. In the boundary regime and most part of the mixed regime, asperity contact friction is dominant while at the end of the mixed and whole part of the fluid-film regime, the traction associated with the fluid plays a dominant role. Comparison between Figs. 4(a) and 4(b) reveals the change in the behavior of asperity contact and hydrodynamic friction coefficient. In the boundary- and mixed-lubrication regimes, an increase in the applied load results in a greater asperity contact and friction coefficient rise. However, in the hydrodynamic regime, the trend is the opposite: An increase in the applied load causes a decrease in the film thickness and friction coefficient decreases. As the friction coefficient approaches to the end of the mixed regime, the contribution of hydrodynamic film in carrying the applied load becomes more pronounced. Therefore, due to the decrease in the hydrodynamic friction coefficient, the total friction coefficient will be reduced. This means that there is a point in the Stribeck curve in which the effect of a change in the applied load on the total friction coefficient changes. It is important to note that thermal effects are neglected.

4.4 Effect of Oil Inlet Viscosity. In Fig. 5 simulations correspond to three different lubricant inlet viscosities. It is clear that the changes in viscosity do not appreciably influence the friction coefficient in the boundary regime since friction is influenced largely by the lubricant’s chemical composition.

Simulation results reveal how the behavior of friction coefficient in the mixed regime changes due to changes in the lubricant viscosity. The lower the viscosity, the greater is the possibility of metal-to-metal contact and, in turn, the higher friction coefficient becomes. In the full-lubrication regime, the friction coefficient is directly related to the viscosity. For a lower viscosity, the lubricant shear stress or friction coefficient is lower. Thus, the change in the viscosity has a different influence in different regimes. It is important to note that depending on the operating condition, the selection of the proper lubricant is important for lowering energy losses. It can be seen that the transition from mixed- to full-film-lubrication is postponed for a lubricant with lower viscosity.

4.5 Effect of Surface Roughness. In this section, the behavior of friction coefficient is predicted by investigating different surface roughnesses. There are three different parameters corresponding to the surface roughness: n, density of asperities; \( \beta \), average radius of asperities; and \( \sigma_r \), standard deviation of asperities. Simulations of different roughness parameters indicate that the change in standard deviation of asperities has a significant effect on the friction coefficient behavior. Simulation results for
different standard deviations of surface roughness are presented in Fig. 6. For a rough surface, the film thickness required to fill out the separation between the asperities is higher. Therefore, under the same operating conditions, there will be more metal-to-metal contact for a rougher surface. This translates to higher values for the friction coefficient.

The results clearly indicate that the velocity in which transition from mixed- to full-lubrication regimes occurs is higher for a rougher surface, as intuitively expected. Since the influence of asperity contact is ignored in driving full-film-lubrication’s equations, there is no difference in the total friction coefficient for different roughness values.

Figures 7(a) and 7(b) show the influence of surface roughness on the hydrodynamic and asperity contact scaling factors, respectively. It is clear that the hydrodynamic scaling factor decreases as the speed increases; this translates to an increase in contribution of hydrodynamic film in supporting the applied load while the reverse is true for the scaling factor associated with the asperity contact. Figures 7(a) and 7(b) reveal that an increase in the surface roughness decreases the asperity scaling factor and increases the hydrodynamic scaling factor.

Figure 8 shows the influence of other roughness parameters, \( n \) and \( \beta \), on the behavior of friction coefficient. For a surface with a higher value for the surface roughness parameter \( \Phi \), the total friction coefficient is higher in the mixed-lubrication regime. Figure 8 reveals that under the conditions simulated, 25% increase in \( \Phi \) may result in up to 12.5% increase in the total friction coefficient in the mixed-lubrication regime. However, for a fixed standard deviation of asperities \( \sigma_s \), the influence of increase in the asperities’ density \( n \), is dominant compared to the increase in the average radius of asperities \( \beta \). This behavior is depicted in Fig. 7. It is important to note that under the conditions simulated, the lift-off speed does not change significantly due to a constant value for \( \sigma_s \).

5 Lift-Off Speed

The speed at which transition from mixed- to full-film-lubrication occurs is often referred to as the lift-off speed. This is the speed at which the friction coefficient attains its minimum value in the Stribeck curve. A series of simulation is conducted to investigate the influence of different input parameters on the lift-off speed. The influences of different operating conditions are lumped in the nondimensional parameters defined in the formulation of the problem. Using the simulation data in the regression analysis, the following equation is derived for the nondimensional lift-off speed:

\[
U_{\text{lift-off}} = 3.9053 \times 10^{-9} \eta_0^{0.01684} G^{-0.8247} \mu^{1.4953} \tag{33}
\]

where \( U_{\text{lift-off}} = \eta_0^{\Phi_{\text{lift-off}}}/E' R \) and \( W, G, \) and \( \mu \) are nondimensional load, material, and roughness parameters, respectively, defined in Sec. 2.

Given specific input parameters, lift-off speed \( \eta_0^{\Phi_{\text{lift-off}}} \) can be easily estimated. It is important to note that this curve fit is only valid within the specific range of given nondimensional parameters listed in Table 3.

Based on Eq. (33), assuming other parameters fixed, any decrease in the fluid inlet viscosity causes an increase in the lift-off speed. Physically, a decrease in the fluid inlet viscosity translates...
to more metal-to-metal contact and higher friction coefficient; this delays the transition from mixed- to full-film-lubrication, which means a high value for the lift-off speed. This is revealed in Fig. 5 comparing Strubeck curve for different oil inlet viscosities. Similarly, an increase in the load parameter will cause a slight increase in the lift-off speed. Figure 6 shows the differences in Stribeck curve for different surface roughness. It is clear that for a rougher surface the transition from mixed to full-film regime will occur at a higher speed. This is in agreement with the proposed relation for lift-off speed; nondimensional surface roughness parameter \( \mu \) will increase due to an increase in the surface roughness, which results in an increase in the lift-off velocity. Some numerical examples are presented in Table 4 and show the comparison of lift-off speed using the proposed relation and full simulations’ result. Hundreds of simulations were performed to test the accuracy of Eq. (33) in comparison to full simulations, and the average error was only 0.76%. However, there existed isolated cases with large errors, caused by curve fitting, as shown in Table 4. These are typically when dealing with high \( G \) and midrange of \( \mu \) values. Nevertheless, Eq. (33) is thought to be useful for quick estimation of the lift-off speed.

### 6 Conclusions

A simple, but realistic model is proposed to predict the behavior of friction coefficient in point contact. The behavior of friction coefficient, when plotted as a function of velocity, reduces to what tribology literature refers to as Strubeck curve. The load-sharing concept is applied to simplify the problem of sliding lubricated contact to dry contact of two spheres and lubricated contact of two smooth spheres. For the contact of two rough spheres, a set of function curve fits has been developed, which can be used to easily predict the load-carrying capacity associated with the asperities. The simulation results are in good agreement with the published experimental data. A parametric study is conducted investigating the influence of different parameters in the behavior of friction coefficient. Parametric study of the model can be useful for design purposes predicting the friction coefficient behavior depends on the operating condition. The proposed model can be used in the design of control algorithm in real-time applications because it is very efficient in terms of computational time and does not encounter numerical stability problems. Based on a series of simulation results under various operating conditions, a relation is proposed to predict the lift-off speed in sliding lubricated point contact with provision for surface roughness.

### Nomenclature

- \( A_H \) = contact area of the fluid, \( \text{m}^2 \)
- \( A_c \) = area of an individual asperity in contact, \( \text{m}^2 \)
- \( a \) = radius of point contact, \( \text{m} \)
- \( c_p \) = constant in Eq. (12), \( 1.962 \times 10^8 \), Pa
- \( D \) = ratio of reduced radii of curvature of mating surfaces, \( =R_x/R_y \)
- \( d_d \) = distance between mean line of asperities and mean line of surface, \( \text{m} \)
- \( E \) = modulus of elasticity, Pa
- \( E' \) = equivalent modulus of elasticity, \( =2[(1-v_1^2)/E_1+(1-v_2^2)/E_2] \), Pa
- \( F_f \) = total friction coefficient
- \( F_{f,H} \) = hydrodynamic friction force, \( \text{N} \)
- \( F_{f,c} \) = asperity contact friction force, \( \text{N} \)
- \( F_c \) = asperity interacting force, \( \text{N} \)
- \( F_H \) = hydrodynamic lifting force, \( \text{N} \)
- \( F_T \) = applied load, \( \text{N} \)
- \( f \) = total friction coefficient
- \( f_c \) = average asperity friction coefficient
- \( h \) = film thickness, \( \text{m} \)
- \( h_c \) = central film thickness, \( \text{m} \)
- \( k \) = aspect ratio of the contact ellipse, \( =a_x/a_y \)
- \( N \) = number of asperities in contact
- \( n \) = density of asperities, \( 1/\text{m}^2 \)
- \( p_c \) = contact pressure of an asperity in the center of point contact, \( \text{Pa} \)
- \( p_m \) = mean pressure of Hertzian contact, \( \text{Pa} \)
- \( p_H \) = hydrodynamic pressure, \( \text{Pa} \)
- \( p_c \) = asperity contact pressure, \( \text{Pa} \)
- \( p_T \) = total pressure, \( \text{Pa} \)
- \( p_0 \) = maximum Hertzian pressure for point contact, \( \text{Pa} \)
- \( R \) = reduced radius of curvature, \( =1/(1/R_1+1/R_2) \), \( \text{m} \)
- \( u \) = effective velocity, \( \text{m/s} \)
- \( Z \) = pressure-viscosity index
- \( \alpha \) = pressure-viscosity coefficient, \( 1/\text{Pa} \)
- \( \beta \) = average radius of asperities, \( \text{m} \)
- \( \beta_0 \) = slope of limiting shear stress-pressure relation
- \( \gamma_1 \) = hydrodynamic scaling factor
- \( \gamma_2 \) = asperity contact scaling factor
- \( \gamma \) = shear rate, \( 1/s \)
- \( \psi \) = dimensionless speed parameter
- \( \eta \) = dynamic viscosity, \( \text{Pa s} \)

### Table 3: Input parameters for parametric study simulations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W )</td>
<td>Load parameter</td>
</tr>
<tr>
<td>( G )</td>
<td>Material parameter</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Roughness parameter</td>
</tr>
<tr>
<td>( U_{v_{\text{lift-off}}} )</td>
<td>Velocity parameter</td>
</tr>
</tbody>
</table>

### Table 4: Lift-off speed (Simulations versus Eq. (33))

<table>
<thead>
<tr>
<th>( W \times 10^6 )</th>
<th>( G )</th>
<th>( \mu )</th>
<th>( U_{v_{\text{lift-off}}} \times 10^{11} ) Eq. (33)</th>
<th>( U_{v_{\text{lift-off}}} \times 10^{11} ) simulation</th>
<th>Abs. residual</th>
<th>Error percentage (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.67</td>
<td>2250</td>
<td>3.58</td>
<td>3.6978</td>
<td>3.3813</td>
<td>0.3165</td>
<td>9.36</td>
</tr>
<tr>
<td>6.67</td>
<td>3000</td>
<td>3.58</td>
<td>2.9167</td>
<td>3.7152</td>
<td>0.7984</td>
<td>21.49</td>
</tr>
<tr>
<td>2.67</td>
<td>1410</td>
<td>3.57</td>
<td>5.3484</td>
<td>5.3088</td>
<td>0.0396</td>
<td>0.746</td>
</tr>
<tr>
<td>1.3699</td>
<td>2058.6</td>
<td>3.57</td>
<td>3.8708</td>
<td>3.8846</td>
<td>0.0157</td>
<td>0.355</td>
</tr>
<tr>
<td>20.4762</td>
<td>1974</td>
<td>2.9814</td>
<td>3.1930</td>
<td>3.1857</td>
<td>0.0073</td>
<td>0.2306</td>
</tr>
<tr>
<td>12.8571</td>
<td>1974</td>
<td>2.9</td>
<td>3.17</td>
<td>3.168</td>
<td>0.0035</td>
<td>0.103</td>
</tr>
<tr>
<td>5.7143</td>
<td>1974</td>
<td>3</td>
<td>3.205</td>
<td>3.2</td>
<td>0.003</td>
<td>0.11</td>
</tr>
<tr>
<td>1.3699</td>
<td>2058</td>
<td>3.6</td>
<td>3.876</td>
<td>3.8708</td>
<td>0.004</td>
<td>0.117</td>
</tr>
<tr>
<td>2.1053</td>
<td>1786</td>
<td>3</td>
<td>4.3554</td>
<td>4.38</td>
<td>0.028</td>
<td>0.64</td>
</tr>
<tr>
<td>10</td>
<td>1974</td>
<td>2.99</td>
<td>3.1571</td>
<td>3.154</td>
<td>0.002</td>
<td>0.075</td>
</tr>
<tr>
<td>1.8182</td>
<td>2068</td>
<td>3.8</td>
<td>3.8393</td>
<td>3.874715</td>
<td>0.035</td>
<td>0.9224</td>
</tr>
</tbody>
</table>
\[ \eta_0 = \text{inlet dynamic viscosity at zero pressure and room temperature, Pa s} \]
\[ \eta_s = \text{constant in Eq. (12), } 6.315 \times 10^{-5} \text{ Pa s} \]
\[ \Lambda = \text{film thickness parameter, } = h_c/\sigma \]
\[ \Phi = \text{surface roughness parameter, } = \sigma \beta \ln \]
\[ \sigma_s = \text{standard deviation of asperities, m} \]
\[ \tau_{c1} = \text{contact shear stress of an individual asperity, Pa} \]
\[ \tau_{f1} = \text{fluid shear stress, Pa} \]
\[ \tau_{l1} = \text{limiting shear stress, Pa} \]
\[ \tau_{l0} = \text{limiting shear stress at ambient pressure, Pa} \]
\[ \nu = \text{Poisson ratio} \]

**Subscripts**

- \( x \) = \( x \)-direction
- \( y \) = \( y \)-direction
- \( 1 \) = body 1
- \( 2 \) = body 2

**References**


